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A TWO–PHASE MODEL FOR INVISCID FLUID

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We consider the system

$$\partial_t s_\pm + \nabla \cdot (s_\pm u_\pm) = 0, \tag{0.1}$$

$$\partial_t u_\pm + Du_\pm \, u_\pm + \nabla p = 0. \tag{0.2}$$

Here, $s_\pm$ are the relative densities of two phases of ideal fluid, subject to a volume constraint $s_+ + s_- = 1$. Moreover, $u_\pm$ are the corresponding velocity fields, and p is a Lagrange multiplier. It is the formal optimality equation associated to the variational problem

$$\inf \left\{ \int \int s_+ |u_+|^2 + s_- |u_-|^2 dx dt \right\}, \tag{0.3}$$

subject to the transport equation (0.1), and time boundary data for $s_\pm$.

The variational problem has minimizers, but the Cauchy–problem (0.1) - (0.2) is ill - posed. The minimizing solutions to (0.3) exist for a finite time, and the mixing entropy

$$\int s_+ \log s_+ + s_- \log s_- dx \tag{0.4}$$

is convex along these solutions as a function of time.

Reference: http://hss.ulb.uni-bonn.de/2013/3122/3122.htm.